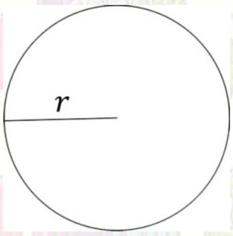
HOSSAM GHANEM

(36) 4.7 Optimization Problems(A)

Circle

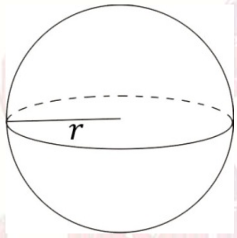
$C = 2\pi r$



$A = \pi r^2$

Sphere

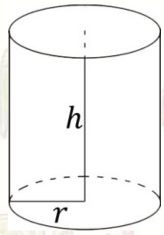
$S = 4\pi r^2$



$V = \frac{4}{3}\pi r^3$

Right cylinder

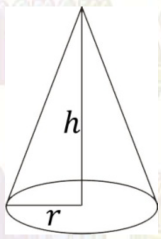
$S = 2\pi r h$



$V = \pi r^2 h$

Right Cone

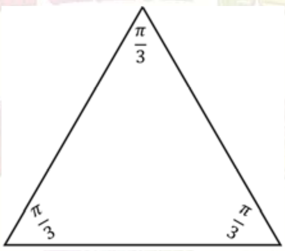
$S = 2\pi r\sqrt{r^2 + h^2}$



$V = \frac{1}{3}\pi r^2$

Equal side triangle

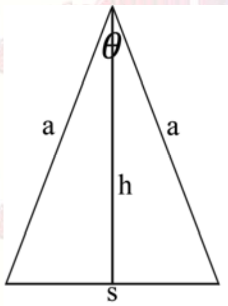
$C = 3a$



$A = \frac{\sqrt{3}}{2}a^2$

Isosceles triangle

$C = 2a + s$

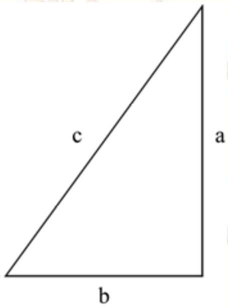


$A = \frac{1}{2}a^2 \sin\theta$

$A = \frac{1}{2}sh$

Right triangle

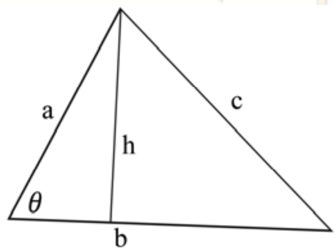
$C = a + b + c$



$A = \frac{1}{2}ab$

Triangle

$C = a + b + c$

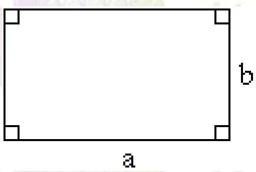


$A = \frac{1}{2}bh$

$A = \frac{1}{2}ab\sin\theta$

Rectangle

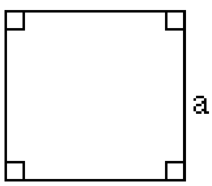
$C = 2(a + b)$



$A = ab$

Square

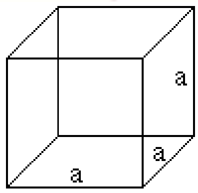
$C = 4a$



$A = a^2$

Coupe

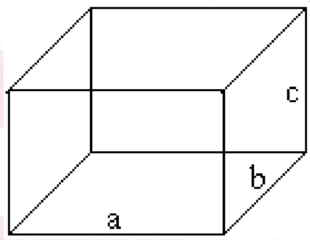
$S = 6a^2$



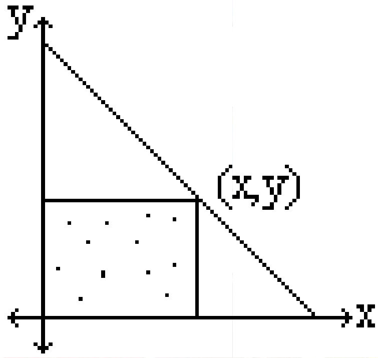
$V = a^3$

Rectangular box

$S = 2ab + 2ac + 2bc$



$V = abc$

Example 1	Find the point $(x\,,y)$ on the line $y + 3x = 3$ so that the area of the shaded rectangle is maximum (see the figure) 

Solution

$y + 3x = 3$

$y = 3 - 3x$

$A = xy$

$A = x(3 - 3x)$

$A = 3x - 3x^2$

$\frac{dA}{ax} = 3 - 6x$

$\frac{dA}{ax} = -6 < 0$ Maxi.

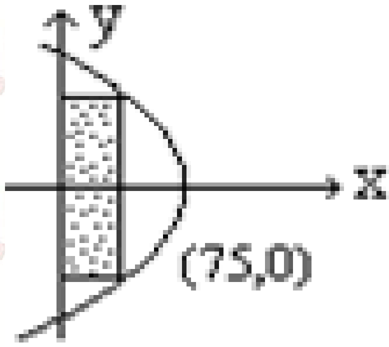
$\frac{dA}{ax} = 0$

$3 - 6x = 0$

$x = \frac{1}{2}$

$y = 3 - \frac{3}{2}$

$y = \frac{3}{2}$ $\quad p\left(\frac{1}{2}, \frac{3}{2}\right)$

Example 2 5 December 1995	Find the dimensions of the rectangle of maximum area that can be inscribed the curve $x = 75 - y^2$ (see figure)	

Solution

$x = 75 - y^2$
$A = 2yx$
$A = 2y(75 - y^2)$
$A = 150y - 2y^3$
$\frac{dA}{ax} = 150 - 6y^2$
$\frac{dA}{ax} = -12y < 0$ MAxi.
$\frac{dA}{ax} = 0$
$150 - 6y^2 = 0$
$y^2 = \frac{150}{6} = 25$
$y = 5$
$x = 75 - 25 = 50$
a Dimension 50 , 10
$p(50, 5)$

Example 3 40 August 7, 2011	(4 Points) Let A be the area of a square , and P be its perimeter .Find the minimum value of $y = A - P$

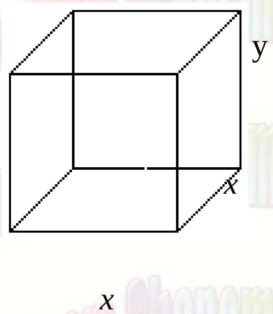
Solution

$y = x^2 - 4x$
$\frac{dy}{dx} = 2x - 4$
$\frac{d^2y}{dx^2} = 2 > 0$ Mini.
$\frac{dy}{dx} = 0$
$2x - 4 = 0$
$x = 2$
$y = 4 - 4(2) = -4$

Example 4 16 June 6, 1996	If box with a square base and an open top is to have a surface area 300 cm^2. Find its maximum volume

Solution

$s = x^2 + 4xy$
$x^2 + 4xy = 300$
$4xy = 300 - x^2$
$y = \frac{300 - x^2}{4x}$
$V = x^2 y =$
$= x^2\left(\frac{300 - x^2}{4x}\right)$
$V = 75x - \frac{1}{4}x^3$
$\frac{dV}{dx} = 75 - \frac{3}{4}x^2$



$\frac{d^2V}{dx^2} = -\frac{3}{2}x < 0$, $x > 0$ Maxi.

$\frac{dV}{dx} = 0$
$75 - \frac{3}{4}x^2 = 0$
$x^2 = 75\left(\frac{4}{3}\right) = (25)(4)$
$x = (5)(2) = 10$ → Maxi
$y = \frac{300 - 100}{40} = \frac{200}{40} = 5$
$V = 100(5) = 500\ cm^3$

Example 5 29 July 25th, 2002	Find two real numbers x and y of minimum product such that $x + y^2 = 12$.

Solution

$p = xy$
$x + y^2 = 12$
$x = 12 - y^2$
$p = y(12 - y^2)$
$p = 12y - y^3$
$\frac{dp}{dy} = 12 - 3y^2$
$\frac{d^2p}{dy^2} = -6y > 0$ if $y < 0$ Mini.
$12 - 3y^2 = 0$
$3y^2 = 12$
$y^2 = 4$
$y = -2$ Mini.
$x = 12 - 4$
$x = 8$
$(x, y) = (8, -2)$

Example 6 30 May 15th, 2003	Find two real numbers x and y such that : $x + y = 16$ and $P = xy^3$ is maximum.

Solution

$x + y = 16$

$y = 16 - x \quad \rightarrow \quad x = 16 - y$

$p = xy^3$

$\quad = y^3(16 - y)$

$\quad = 16y^3 - y^4$

$\frac{dp}{dy} = 16(3)y^2 - 4y^3$

$\frac{d^2p}{dy^2} = 16(6)y - 12y^2$

$\quad = 12y(8 - y)$

$\frac{dp}{dy} = 0$

$16(3)y^2 - 4y^3 = 0$

$4y^2\,(12 - y) = 0$

$y = 0$

$y = 12$

$\left.\frac{d^2p}{dy^2}\right|_{y=12} = 12(12)(8 - 12) < 0$ Maxi.

$x = 16 - 12 = 4$

the numbers are 4 and 12

Example 7 41 7 January 2012	[4 Pts.] Find the point on the line $x - y = 40$ for which $P = x^2 + y^2$ is minimum .

Solution

$x = y + 40$

$P = x^2 + y^2$

$P = (y + 40)^2 + y^2$

$\frac{dP}{dy} = 2(y + 40) + 2y = 4y + 80$

$\frac{d^2P}{dy^2} = 4 > 0$ Mini.

$\frac{dP}{dy} = 0$

$4y + 80 = 0$

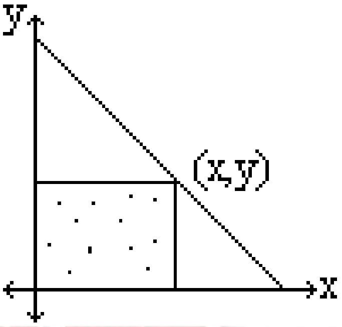
$y = -20$

$x = -20 + 40 = 20$

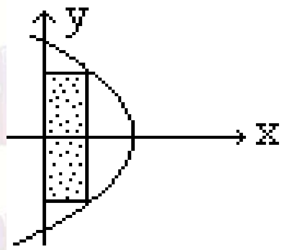
The point (20 , −20)

Homework

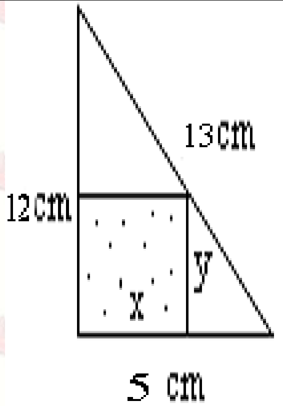
1 Find the point (x , y) on $3y + x = 3$ so that the area of the shaded rectangle is maximum (see the figure)



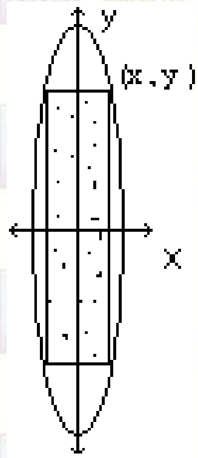
2 Find the dimensions of the rectangle of maximum area that can be inscribed the curve $x = 48 - y^2$ (see figure)



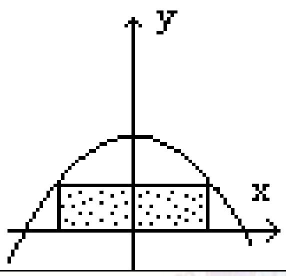
3 A rectangle is be inscribed inside aright triangle of side *5 cm , 12 cm , 13 cm .* Find the dimensions of the rectangle positioned as the figure so that its area is maximum.



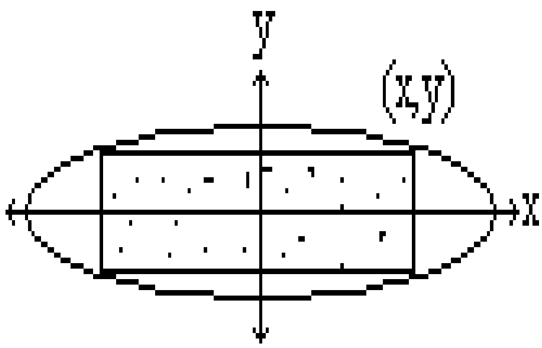
4 Find the dimensions of the rectangle of maximum area that can inscribed the Curve $16x^2 + y^2 = 16$ and having sides parallel to the coordinate axes



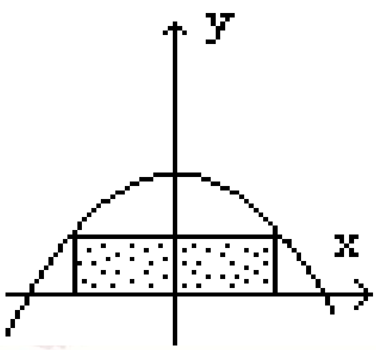
5 Find the point (x , y) on the curve $y = 75 - x^2$ which make the area of The shaded rectangle in the figure maximum



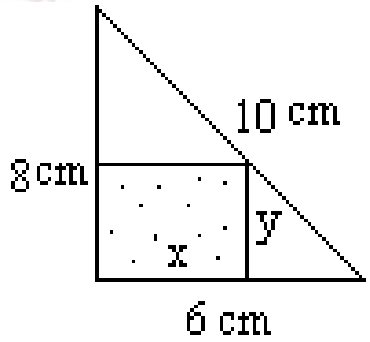
6 3 December 30, 1991
15 February 1996
Find the dimensions of the rectangle of maximum area that can inscribed the curve $x^2 + 4y^2 = 4$ and having sides parallel to the coordinate axes (see the figure)



7 1 January 1990
Find the point (x , y) on the curve $y = 80 - x^2$ which make the area of the shaded rectangle in the figure maximum



Homework

8	8 August 28, 1993 A rectangle is be inscribed inside aright triangle of side *6 cm , 8 cm , 10 cm* Find the dimensions of the rectangle positioned as the figure so that its area is maximum 
9	Find the dimensions of the rectangle area $64\ cm^2$ whose perimeter is minimum
10	If box with a square base and an open top is to have a surface area 48 cm^2. Find its maximum volume
11	11 August 11, 1994 A A wire *20 cm* long is to be cut into two pieces. If each piece is bent into the shape of a square. Where should the wire be cut so that the sum of their areas is minimum?
12	5 July 13, 1992 A rectangular garden of area $75\ ft^2$ is bounded on three sides by a wall costing $8 per ft and on the fourth side by a fence costing $4 per ft. What are the dimensions of the garden for minimum cost.
13	A farmer wants to enclose a rectangular field by using a straight river as a side, it is known that the two opposite sides cost 2 K.D per foot and the other side opposite to the river cots 3 K.D per foot. If he has 800 K.D for the project, what dimensions would enclose a maximum area ?
14	Find the dimensions of the right circular cylinder , with open top and closed bottom of maximum surface area that can be inscribed inside a right circular cone of radius *6 cm*. and height *18 cm*
15	A line having (negative) slop m passes through (2 , 18) and intersects the coordinate axes at (a , 0) and (0 , b) . Find the value of m which makes $a + b$ minimum.
16	Find the dimensions of the rectangle area $81cm^2$ whose perimeter is minimum
17	35 August 15, 2009 Let x and y be two positive numbers whose sum is 4 . Find the values of x and y that minimize the function $P = x^2 + y^2$.
18	37 June 6, 2010 two positive numbers whose product is 100 and whose sum is minimum
19	39 5 June, 2011 [4 pts.] Find the value of $m < 0$ that minimizes the area of the region bounded by the $x -$ axis , the y $-$ axis and the line $y = mx + 5 - 3m$.
20	23 May 26. 2002 Find the minimum value of $S = 9x + 8y^2$, where $x(y^2 + 1) = 2$ and $x \neq 0$. (4 pts.)

17 35 August 15, 2009	Let x and y be two positive numbers whose sum is 4 . Find the values of x and y that minimize the function $P = x^2 + y^2$.

Solution

$x + y = 4$
$y = 4 - x$
$P = x^2 + y^2$
$P = x^2 + (4 - x)^2$
$\frac{dp}{dx} = 2x - 2(4 - x) = 2x - 8 + 2x = 4x - 8$
$\frac{d^2p}{dx^2} = 4 > 0$ Mini.
$\frac{dp}{dx} = 0$
$4x - 8 = 0$
$x = 2$ Mini.
$y = 4 - 2 = 2$
$(x, y) = (2,2)$

20 23 May 26. 2002	Find the minimum value of $S = 9x + 8y^2$, where $x(y^2 + 1) = 2$ and $x \neq 0$. (4 pts.)

Solution

$x(y^2 + 1) = 2$
$y^2 + 1 = \frac{2}{x}$
$y^2 = \frac{2}{x} - 1$
$S = 9x + 8y^2$
$S = 9x + \frac{16}{x} - 8$
$\frac{ds}{dx} = 9 - \frac{16}{x^2} = 9 - 16x^{-2}$
$\frac{d^2s}{dx^2} = \frac{32}{x^3} > 0$ if $x > 0$
$\frac{ds}{dx} = 0$
$9 - \frac{16}{x^2} = 0$
$x^2 = \frac{16}{9}$
$x = \frac{4}{3}$ Mini. , $x = -\frac{4}{3}$ Maxi.
$y^2 = \frac{2}{\frac{4}{3}} - 1 = \frac{6}{4} - 1 = \frac{2}{4} = \frac{1}{2}$
$S = 9\left(\frac{4}{3}\right) + 8\left(\frac{1}{4}\right) = 12 + 2 = 14$